EGT and economics:
I. Optimality principles and models of behaviour dynamics

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Introduction
Classical game theory

Game theory is widely used for description and analysis of economic players' behavior in microeconomics, public sector economics, political economics and other fields of economic theory.

General ideas of classical game-theoretic analysis:

●**Game in normal form** (as a model of players interaction)

-Several participants (players)

- Several possible strategies for each player

-Payoff functions

●**Principle of Nash equilibrium** (as a method to define agents strategies during their interaction)

-Nash equilibrium is a basic concept of game theory. According to this optimal outcome of a game is one where no player has an incentive to deviate from his or her chosen strategy after considering the choices of other players.

●**Principle of elimination of dominated strategies**

-Strategy is called dominated if there exists an alternative strategy which provides a greater gain no matter what strategies are chosen by the other players

-Domination principle means that rational players will not use dominated strategies

-Dominance elimination can be made iteratively

Evolutionary game theory: considered problems

- 1. Correspondence of real behavior of economic agents to Nash equilibrium and dominance elimination principles
  - Typically the search of Nash equilibrium and sets of nondominated strategies deliveres rather sophisticated math tasks. It is necessary to know all sets of strategies and payoff functions for their solution (see models of Cournout and Bertrand economical competition) .
  - A usual participant of such interaction has precise information only about his own strategies and payoff function and often doesn't know about mentioned decision-making principles
  - Why should we expect that his behavior will be relevant to the principles?
  - Justification by means of models of adaptive and imitative behavior (MAIB)
    - These models show that convergence to Nash equilibrium and dominance elimination proceed from general properties of evolutionary, adaptative and imitative mechanisms of behavior formation
    - In this case complete information awarness and rationality in choosing strategies are not required. It suffices to compare the payoffs for current behavior strategy and chosen alternative.
    - Definition of evolutionary stable strategy and its relation to Nash equilibrium.

- 2.How to determine players utility functions for particular interactions? Standard approach: use concepts of particular sciences.
  - In economics: 'homo economicus‘ concept (P.Samuelson)
    - In the role of a producer: profit maximisation
    - In the role of a consumer: maximisation of the demand utility
    - Discrepancy to real behavior: russian labour market.

    Alternative: consider evolution of preferences (L. Samuelson)
  - Model of evolutionary mechanisms natural selection
    - In this model a society of interacting populations with different evolutionary mechanisms is considered
    - Analysis of this model shows that if replication is in the set of competing mechanisms then behavior dynamics in the society is coordinated with maximization of individual fitness (Ch. Darwin). What about human populations?
- 3.Do individuals adjust to other interests or extensively influence payoff functions of other players? (agressive advertising, drug distribution)

Population game

Population game is a static model of interaction in a large homogenuous group of individuals.

This concept is an analog of a normal form game in classic game theory, so general noncooperative optimality principles are summarized:

- *Nash equilibrium*
- *Dominance solution*
- Idea of *evolutionary stable strategy* is added

Formally a **population game** G is a set of parametres

G = $< J, f_j(\pi,\omega), j \in J, \pi \in \Pi, \omega \in \Omega >$, where

- J is the set of players strategies
- $\Pi = \left\{ \pi \middle| \pi_j \geq 0, \sum_{j \in J} \pi_j = 1 \right\}$ standart simplex
- $f_j(\pi,\omega)$ is the payoff function for players that use strategy *j under* strategy distribution

and other parametres of the model ω (e.g. total population size and environmental conditions). For social populations, the payoff function usually corresponds to consumer utility, income or profit. In this paragraph the function is exogenous.

Example of population game (M.Smith)

Consider pair competitive contests for one resource. Individuals of a population try to find the desired object (food, lodging or a female). Some of them get it without collision, others conflict in pairs, where one of them is the *owner* α and another one is *invader* β

- J^{α}, J^{β} - the sets of strategies in each role
- $\phi^{\alpha}_{j^{\alpha} j^{\beta}}, \phi^{\beta}_{j^{\alpha} j^{\beta}}$ - profits of individuals if α choses $j^{\alpha} \in J^{\alpha}$ and β - $j^{\beta} \in J^{\beta}$
- $\lambda(N)$ - probability of collision. It doesn't depend on startegies and is determined by the size of population N
- ϕ^{0} - profit of individuals avoided collision
- Strategy of individual is a pair $j = (j^{\alpha}, j^{\beta})$, where $j^{\alpha} \in J^{\alpha}$ and $j^{\beta} \in J^{\beta}$. It is a rule of behavior choice according to the role.
- Function $f_j(\pi, N)$ shows the average payoff of individuals with strategy j

Let $p_\alpha(\pi)$ and $p_\beta(\pi)$ denote distributions over alternatives for the both roles α and β which correspond to strategy **π** distribution. Then

$$p_{j^\alpha}(\pi) = \sum_{j^\beta \in J^\beta} \pi_{j^\alpha j^\beta} \quad , \quad p_{j^\beta}(\pi) = \sum_{j^\alpha \in J^\alpha} \pi_{j^\alpha j^\beta}$$

And for strategy $i = (i^\alpha, i^\beta)$

$$f_{i^\alpha i^\beta}(\pi, N) = (1 - \lambda(N))\phi^0 + \frac{1}{2}\lambda(N)\Big(\sum_{j^\beta \in J^\beta} \phi^\alpha_{i^\alpha j^\beta} p_{j^\alpha}(\pi)\Big)$$

Consider also the situation where **players don't distinguish their roles**. Then the set of strategies is the set of alternatives: $J = J^\alpha = J^\beta, \pi = (\pi_j, j \in J), \phi^\alpha_{ij} = \phi_{ij}, \phi^\beta_{ij} = \phi_{ij}$

$$\overline{f_i}(\pi) = \sum_{j \in J} \pi_j \phi_{ij}, f_i(\pi, N) = (1 - \lambda(N))\phi^0 + \lambda(N)\overline{f}_i(\pi)$$

In this case game G is equivalent to the game $\overline{G} = \left\langle J, \overline{f_i}(\pi), i \in J, \pi \in \Pi \right\rangle$

The set of behavior alternatives and individual payoffs may not depend on the role in the previous model too. However, behavior models for those similar situations are completely different.

Main static optimality principles

Nash equilibrium for the populational game *G* is a distribution ***π**** such that any startegy which is used with positive frequence is an optimal reply to the given distribution under any parameter *ω, i.e.*

$$\forall \omega \in \Omega, \forall j \in J(\pi_j^* > 0) \Rightarrow j \in Arg \max_{i \in J} f_i(\pi^*, \omega) \tag{2.1}$$

Let payoff functions in game *G* be expansible, i.e. $f_j(\pi,\omega) = a(\pi,\omega)\overline{f_j}(\pi) + b(\pi,\omega) > 0$ as in the model of pair contests. Note that a part of the payoff function that depends on the strategy chosen by a player is independent of the parameter *ω*. Then (2.1) is equivalent to the following condition, which doesn't include the parameter *ω:*

$$\forall j \in J : \pi_j^* > 0 \Rightarrow j \in Arg \max_{i \in J} \overline{f_i}(\pi^*)$$

The concept of Nash equilibrium is the best-known optimality criterion used in strategic behavior modeling. However, it is known from analysis of dynamic models, that among Nash equilibrium there can exist unstable states, that are not realized in practice. For this reason we also consider **stronger optimality criteria.**

Evolutionary stable strategy (ESS) for the populational game *G* is a distribution ***π****
such that $\forall \omega \in \Omega \forall \pi \neq \pi^* \exists \overline{\lambda}(\pi) \in (0,1) : \forall \lambda \in (0, \overline{\lambda}(\pi))$

$$f_{\pi^*}(\lambda\pi + (1-\lambda)\pi^*, \omega) > f_{\pi}(\lambda\pi + (1-\lambda)\pi^*, \omega)$$

where $f_\pi(\pi', \omega) = \sum_{j \in J} \pi_j f_j(\pi', \omega)$ is the average payoff for mixed strategy or distribution π when individuals in the population are distributed in pure strategies according to π'.

The concept of ESS can be interpreted in the following way. Let a small group of 'mutants' with strategy distribution π be implemented in a population π. If distribution π* is evolutionary stable then the implemented group can't survive in the population, because its average fitness is less then the fitness of initial strategy π*.*

Any ESS is a Nash equilibrium. If π is not an equilibrium then mutants with pure strategy that is a best reply to π have grater payoff than the average payoff for the main population. This **statement is correct if the share of one individual in the population is negligible**, i.e. any change of his strategy doesn't influnce payoff ordering. Otherwise it is necessary to revise the ESS concept (Schaffer,1989).

For a symmetrical game in normal form with n players, set of strategies S and payoff function $f_j(s_j, s_{J\setminus j})$, ESS is defined as a symmetric situation $s_j = s$ such that any strategy change by any player doesn't make his profit more than the profit of other players with the former strategy. *I.e. a mutant doesn't have any profit benefits.*

Such ESS can be not a Nash equilibrium. In particular, in the game which correspond to symmetric Cournout oligopoly, players use 'market power' at the Nash equilibrium and realise lower production volumes in comparison with the competitive equilibrium, while the ESS corresponds to the competitive equilibrium.

Strict equilibrium of population game G is distribution π^* such that all players use one strategy, which is the best reply to itself:

$$\exists \varepsilon > 0, \exists j \in J : \pi_j^* = 1, \forall i \neq j, \forall \omega \in \Omega \quad f_j(\pi^*, \omega) > f_i(\pi^*, \omega) + \varepsilon$$

Note, that **any strict equilibrium is an ESS**, even for groups with sufficiently large finite size.
Selten (1988) showed that there are no ESS except for strict equilibria for random contests with role asymmetry of players ('owner' — 'invader').

For general payoff functions $f_i(\pi\ ,\omega)$ **Nash equilibrium may not exist**. In other classes of games there are a lot of equilibria, some of them are unstable. In this case another optimality principle is appealing — **domination** that is also relative to the Darwinian principle of natural selection.

Strategy *j* dominates strategy *i* $j \succeq i$ on the set of distributions $\Pi' \subseteq \Pi$ if for any distribution over strategies $\pi \in \Pi'$ the strategy *j* **provides a greater gain** than strategy i $(\exists \varepsilon \geq 0 : \forall \omega \in \Omega, \forall \pi \in \Pi' \Rightarrow f_j(\pi, \omega) \geq f_i(\pi, \omega) + \varepsilon)$

Set $J' \subseteq J$ is called a **dominating set** if it can be obtained by iterative **exclusion of dominated strategies**, i.e. there exists integer T >1 such as

$$J' = J_T \subset J_{T-1} \subset \ldots \subset J_1 = J, where \forall k \in \{1, \ldots, T-1\}, \forall i \in J_k \setminus J_{k+1} \exists j \in J_{k+1} :$$

$$j \succeq i \quad on \quad \Pi_k = \{\pi \in \Pi, \pi_j = 0, \forall j \notin J_k\}$$

The described procedure for iterative elimination of dominated strategies can be considered as a quasi-dynamic model of behavior microevolution within a population. Indeed, this procedure describes a sequential reduction of the set of strategies used by players: at each stage, more efficient (better fitted) strategies are substituted for less efficient ones.

If ε>0 then strategy *j* **strictly dominates** strategy i $(j \succ i)$, J' **is a strictly dominating set**.

Concepts of domination by a mixed strategy and a set domination in mixed strategies are similar

Search of Nash equilibrium and dominating sets of a population game is generally a sophisticated optimization problem. For random pair contests it is possible to reduce them to the known computational tasks for appropriate **bimatrix games** .

Proposition2.1 Distribution π^* is a Nash equilibrium of game $\overline{G}$ such that $\overline{f}_i(\pi) = \sum \pi_j \phi_{ij}$ and competitors don't differ conditions if and only if (π^*, π^*) is Nash equilibrium in mixed strategies of symmetrical bimatrix game $\Gamma = \left\langle (\phi_{ij})_{i,j \in J}, (\phi_{ji})_{i,j \in J} \right\rangle$ i.e $\forall j \in J(\pi_j^* > 0) \Rightarrow j \in Arg \max\limits_{i \in J} \overline{f}_i(\pi)$

Proposition2.2 Distribution π^* such that $\pi_s^* = 1$ is strict equilibrium of game $\overline{G}$ if and only if for any $i \neq s$ $\phi_{22} > \phi_{i2}$,i.e. *(s,s)* is strict symmetric Nash equilibrium of game Γ in pure strategies.

Proposition2.3 Strategy *s* dominates strategy *r* ($s \succeq r$) in game $\overline{G}$ if and only if $s \succeq r$ in game Γ, i.e $\phi_{sj} \geq \phi_{rj}$ for any $j \in J$

Proposition2.4 Distribution $\pi = (\pi_{j^\alpha j^\beta})$ is Nash equilibrium of game G for assymetric pair contests if and only if $(p^\alpha(\pi), p^\beta(\pi))$ is Nash equilibrium in mixed strategies of game Γ=
$= \left\langle (\phi^\alpha_{j^\alpha j^\beta}), (\phi^\beta_{j^\alpha j^\beta}) \right\rangle$

- So, **for any random pair contests Nash equilibrium of population game correspond to Nash equilibrium of bimatrix game** that describes pair interaction
- Analogue relation exists for random interactions with larger number of participants when a separate local interaction is characterized by the game of n players. The results are easily generalized for the case of interpopulation collisions when individuals from different populations or social groups ('predator-prey', 'employer-employees' etc) play different roles.

  The main condition of the correspondence is *non-correlation of distribution in interacting groups with players' strategies*

Model of adaptive-imitative behavior (MAIB)

In which cases adeptive-imitative mechanisms form population behavior that corresponds to Nash principle and principle of dominated strategy elimination?

Let population game $G = \langle J, f_j(\pi) \rangle$ describe interaction of population individuals that happens continiously, at every moment of time.

- The number of individuals and the external factors are fixed(f_j do not depend on ω)
- With intensity $r_j = r_j(f(\pi), \pi)$ which depends on current player distribution over strategies π and current payoff vector $f(\pi) = (f_j(\pi), j \in J)$ a player with strategy j turns into 'adaptive' status where reconsiders his behavior
- In adaptive status player with startegy j chooses i as alternative with probability $q_{ji} = q_{ji}(f(\pi), \pi)$
- Current and alternative strategies are compared. If alternative strategy i is better then initial strategy j (i.e. gives individual the greater distribution under this distribution over strategies) then the player changes his strategy to i with probability $\gamma_{ji} = \gamma_{ji}(f(\pi), \pi)$

Then $r_j\pi_j\sum_{i:f_i>f_j} q_{ji}\gamma_{ji}$ is the average share of players who change their strategy *j* to strategy from set $\{i|f_i > f_j\}$ at time t, $\sum_{i:f_i<f_j} r_i\pi_i q_{ji}\gamma_{ji}$ - the share of players who change their strategy from set $\{i|f_i < f_j\}$ to *j*.

So, equations of behavior dynamics look like $\dot{\pi}_j = -r_j\pi_j\sum_{i:f_i>f_j} q_{ji}\gamma_{ji} + \sum_{i:f_i<f_j} r_i\pi_i q_{ji}\gamma_{ji}$ (2.2)

Functions r_j, γ_{ji}, q_{ji} are such that $\forall j \in J \quad r_j \geq 0; \forall i, j \in J \quad \gamma_{ji} \geq 0, q_{ji} \geq 0; \forall j \in J \sum_{i\in J} q_{ji} = 1$

Mentioned conditions guarantee that path $\{\pi(t,\pi^0)\}$ doesn't come out set Π at any moment of time *t* and with any initial distribution π^0

MAIB. Examples.

Example 1. Let the intensity of status change to adaptive be constant. Alternative strategy is chosen by means of random imitation. And the probability of changing current strategy to alternative is proportional to the difference between the corresponding payoff functions. So,

$$r_j(f(\pi),\pi)\equiv r, q_{ji}(f(\pi),\pi)\equiv \pi_i, \gamma_{ji}(f(\pi),\pi)\equiv \gamma(f_i(\pi)-f_j(\pi))$$

And (2.2) looks like

$$\overset{\bullet}{\pi}_j = r\gamma\pi_j(f_j(\pi)-\sum_{i\in J}\pi_i f_i(\pi), j\in J$$

This system is an analogue of autonomus continuous model of replicator dynamics (see part 3)

Example 2. Alternative strategy is chosen with equal probabilities from the set of possible strategies, i.e.

$$q_{ji}(f(\pi),\pi)=1/|J|$$

This example illustrates the mechanism of individual adaptation when each player knows the whole set of possible strategies, and adaptation happens according to current payoff values. Adaptation doesn't depend on behavior of other population individuals.

It is evident that there are many different MAIBs. The following theorems reveal relation **MAIB stable states and solutions of the corresponding population game**. Note that **any Nash equilibrium of population game G is a steady state of MAIB.**

Relation between MAIB stable states and population game solutions

Denote $J(\pi) = Arg \max_{k \in J} f_k(\pi)$ as a set of best replies to distribution π .

Theorem 2.1: Let MAIB meet the following conditions 1), 2) and 3) or 3') :

1)For any $j \in J$ and any $r_j > 0$

(intensity of changing status to adaptive is positive for all strategies)

2) For any $i, j \in J$ functions γ_{ji} look like $\gamma(f_i(\pi) - f_j(\pi))$ where for any $x > 0, \gamma(x) > 0$

(probability of strategy choice as alternative is function of corresponding payoff difference and is positive funder the positive argument)

3)For any $j \in J, i \in J(\pi)$ $q_{ji} > 0$

(probability of strategy choice as alternative is positive for any strategy that gives maximum payoff under the current distribution over strategies)

3')For any $j \in J, i \in J(\pi)$ $q_{ji} > q\pi_i$ and $q>0$

(for any pure strategy with maximum payoff, probability of this strategy choice as alternative is not less than its share in the population multiplied by some constant)

Then

Any Lyapunov stable point π^* of system (2.2) is Nash equilibrium of population game $G = \langle J, f_j(\pi), j \in J, \pi \in \Pi \rangle$

b) if initial distribution $\pi^0 > 0$ and for path $\{\pi(t, \pi^0)\}$ there exists $\lim_{t \to \infty} \pi(t, \pi^0) = \pi^*$ then π^* is Nash equilibrium, for game G

c) if π^* is a of strict equilibrium for population game G then π^* is an assymptoticaly stable point of system (2.2)

Theorem 2.2: Let MAIB (2.2) meet conditions 1 and 2 of theorem 2.1 and moreover

1.for any $i, j \in J$ $q_{ji} = \pi_i$
(alternative strategy is chosen by random imitation)
2. if $f_j \geq f_i$ then $r_j \leq r_i$
(intensity of changing status to adaptive decreases with the raise of payoff function)
3. $\gamma(x)$ increases monotonically in x
(probability of strategy choice as alternative rises monotonically for payoff remainder)

If $\overline{J}$ is a strictly dominating set of strategies in populational game $G = \left\langle J, f_j(\pi), j \in J, \pi \in \Pi \right\rangle$ then, for any $j \notin J$ and initial distribution $\pi^0 > 0$ $\lim\limits_{t \to \infty} \pi_j(t, \pi^0) = 0$

Notes

•Other variants of such consistency conditions of MAIB dynamics with Nash and dominance decisions (see Samuelson L., Zhang J.(1992) and Weibull (1995)) relate to the concept of monotonous dynamics $\dot{\pi}_j = \pi_j g_j(\pi), j \in J$ s.t. $g_j(\pi) > g_i(\pi) \Leftrightarrow f_j(\pi) > f_i(\pi), \forall i, j \in J, \pi \in \Pi$
At the same time there exist adaptation models that don't meet theorems 2.1 and 2.2. Models of evolutional mechanisms' natural selection considered in the next part explain why we nevertheless should expect coherence of real behavior dynamics with the mentioned optimality principles. Moreover, payoff functions of players are endogeneously defined in the frames of these models.

Replicator Dynamics Model (RDM)

- Population is characterized with a set S of possible strategies
- Strategy distribution of individuals at the current moment is set by vector $\pi = (\pi_s, s \in S)$
- Individuals differ only in behavior strategies, they don't change it during their life, strategy is inherited
- In case of populations with both males an females, each of them should be considered as a separate population
- Genetical mechanism of inheritance: the strategy is determined by genes, connected with sex. Mechanism of imitation: the strategy is defined by imitating behavior of the parent of the same sex.
- The result of the interaction in population during a period of time is characterized for players with strategy s by fertility function $fer_s(\pi, N)$ that determines the average number of offsprings. It is also characterized by survival function $\upsilon_s(\pi, N)$ that determines the probability to survive under distribution π and population size N.

Denote $N_s = \pi_s N$ - the number of those who use strategy s .
Then popultion dynamics $N_s(t), s \in S$, meets the following system:

$$N_s(t+1) = N_s(t) f_s(\pi(t), N(t)) \qquad (3.1)$$

Where $f_s(\pi, N) = fer_s(\pi, N) + \nu_s(\pi, N)$ is a *strategy fitness function*. It formalizes the Darwinian concept of individual fitness.
At first sight a concept of payoff function is inapplicable to this model. Players' strategies are fixed, the are not approaching to smt and don't choose anything. However, the picture changes if we look at strategy distribution dynamics.

The following theorem shows that *behavior assymptotics in such population corresponds to fitness as a payoff function for this population.* Particularly, if for $t \to \infty$ strategy distribution approaches to stationary, then there are only those strategies in population that maximize fitness (corresponds to Darwinian principle of natural selection: only most fitted survive). If in any distribution one strategy fits better than another, then a part of the worst strategy in distribution $\pi(t)$ approaches to 0 while $t \to \infty$ .In this case fitness is at endogenuous utility function of this model.

Relation of Nash equilibria with stable MRD points
Asymptotic stability of ESS
Relation of dominating sets of strategies with behavior dynamics

Theorem 3.1 (on relation of Nash equilibrium with stable points of RDM): Assume that the fitness function f_s is reperesentable in additive form $f_s(\pi,\omega)=a(\pi,\omega)+b(\pi,\omega), a(\pi,\omega)>0$
Then
1) any stable (Lyapunov) distribution π^* of system (3.1) is a Nash equilibrium in population game $G=\langle S, f_s(\pi), s\in S, \pi\in\Pi\rangle$
2) if for a certain path $\{\overline{N}(t)>0\}$ the initial distribution $\overline{N}(0)>0$ and $\exists\lim\limits_{t\to\infty}\pi(t,\overline{N}(0))=\pi^*$ then π^* is a Nash equilibrium of the specified population game

Note *that system (3.1) isn't closed, because its right part also depends on N(t). Conception of stable distribution for such system is formally defined in Bogdanov, Vasin (2002)*

Theorem 3.2 (on asymptotic stability of ESS): Assume that in theorem 3.1 π^* is *evolutionary stable strategy* for population game $\overline{G}$. Then π^* is an asymptotically stable distribution of system (3.1)

Theorem 3.3 (on the relation between dominating sets of strategies and behavior dynamics): Assume that *S* is a strictly dominating set of strategies in the game $G'=\langle S, \ln\overline{f}_s(\pi), s\in S, \pi\in\Pi\rangle$. Then, for any $s\notin S$ and any $\overline{N}(0)>0$ $\lim\limits_{t\to\infty}\pi_s(t,\overline{N}(0))=0$ on the corresponding path of system 3.1

Random imitation

Replicator Dynamics Model describes activity of evolutionary mechanism that provides direct inheritance of strategies by children. In what degree are the stated results depend on *concrete evolutionary mechanism?* Turns out that *it plays a very important role*. Let's consider the **mechanism of random imitation** as an alternative example

This model differs from replicator dynamics only in one point: *new individuals do not inherit strategy, they follow a strategy of randomly chosen adult.* Then the population dynamics are described $N_s(t+1) = N_s(t)v_s(t) + \sum_r N_r(t) fer_r(t) \frac{N_s(t)v_s(t)}{\sum_r N_r(t)v_r(t)}, s \in S$

Such system dynamic corresponds to payoff function $v_s(t)$ in the sense of theorems 3.1-3.3. Thus, viability turns out to be an endogenous payoff function of individuals in the corresponding dynamical process.

Proceeding from the previous example it seems that we have exchanged arbitrariness in the choice of payoff functions for arbitrariness in the choice of evolutionary mechanisms. However, actual evolutionary mechanisms are subject to natural selection. Only the most efficient mechanisms survive in the process of competition.

Model of evolutionary mechanisms competition

Consider the corresponding *model of a society that includes several populations* that differ only in their evolutionary mechanisms.

- Individuals of all populations interact and do not distinguish population characters in this process. Thus, the evolutionary mechanism of an individual is an unobservable characteristic.
- Fertility and viability functions describe the outcome of the interaction for each strategy and depend on the total distribution over strategies and the size of the society.
- The set of strategies *S* and the functions are the same for all populations.

Denote

- L – the set of populations
- N_l - the size of population *l*
- *N* – the total population size
- $\pi^l = \{\pi_s^l, s \in S\}$ - distribution over strategies in population *l.*

Then the total distribution over strategies is $\pi = \sum \frac{N^l}{N} \pi^l$

Assume that operator Φ^l corresponds to the evolutionary mechanism of population *l* and determines the dynamics of distribution π^l .

(For example, in one population it is replication dynamics, in an other - random imitation, and so on. Particularly, dynamics may relate to maximization of some payoff function)

Then the dynamics of the society are governed by equations

$$N^l(t+1) = N^l(t)\sum_s \pi_s^l(t) f_s(\pi(t), N(t)) \tag{3.2}$$

$$\pi^l(t+1) = \Phi^l(\pi^k(t), N^k(t), k \in L), l \in L$$

Theorem 3.4. Let there exists a population of replicators in the society. Then the total distribution $\pi(t)$ over strategies meets the following analogs of theorems 3.1 and 3.2:

1) any stable distribution π of system (3.2) is a Nash equilibrium of the population game $\overline{G} = \left\langle S, \overline{f}_s(\pi), s \in S \right\rangle$

2) if for path $\left\{\overline{N}(t)\right\}$ initial distribution $\overline{N}(0) > 0$ and $\lim_{t\to\infty} \pi(\overline{N}(0), t) = \pi^*$ then π^* is a Nash equilibrium of the population game G for path

3) if π is a strict equilibrium of the game *G* then π is an asymptotically stable distribution for system (3.2).

Thus, the evolutionary mechanisms selection model confirms that individual fitness is an endogenous utility function for self-reproducing populations.

- The idea of the proof for propositions 1 and 2 is quite easy: if stationary distribution over strategies is not Nash equilibrium of fitness function then nothing can prevent expansion of replicators that use the best reply strategy. That is a contradiction to its stability

- Generalization of theorem 3 for elimination of dominated strategies is possible under more strict assumptions on the variety of evolutionary mechanisms. For any evolutionary mechanism Φ^l and a pair of strategies *s,r*, let us call an *s,r*-substitute of mechanism Φ^l a mechanism $\Phi^l_{s,r}$ such that for strategies other than *s and r* the shares of individuals who apply these strategies change as under mechanism Φ^l, except that instead of strategy *s* they always play *r*. According to (Vasin, 1995), if for any *s,r,l* the set of mechanisms includes all possible substitutes of $\Phi^l_{s,r}$ then $\pi_s(t) \to 0$ as $t \to 0$ for any strictly dominated strategy *s*.

- The results of this section are formulated for a homogenuous population, without taking sex or age in consideration. It is easily generalized for populations with such structures. Fitness analogue in this case is a rate of balanced growth of the population, it is determined by the Frobenius number of the Leslie matrix (see Semevskiy, Semenov, 1982)

Conclusion

- Models and results of evolutionary game theory show that behavior evolution in a self-reproducing population corresponds to well-known optimality principles — Nash equilibrium and elimination of dominated strategies
- Endogenously formed payoff function corresponds to Darwinian concept of individual fitness

Problems

- However in biological and social populations cooperative and altruistic behavior are well-known. It seems that they don't correspond to optimization of individual fitness
- Problem of stability of mixed equilibrium, where more than one pure strategy is used with positive probability. *This problem appears in case of interpopulation interaction where payoff for one population depends on distribution over strategies in another population. It also appears in case of random contests with role asymmetry between participants. For such games mixed Nash equilibria are never evolutionary stable and strict equilibria may not exist. So, sufficient conditions of stability don't work*
- Relevance of mentioned evolutionary models to social populations. Superindividual is a self-reproducing structure that uses human population as a source for its reproduction. It can influence behavior dynamics in this population

II. Stability of equilibrium. Pecularities of social behavior evolution.

Stability problem for mixed equilibrium

Consider game Γ of two populations with sets of strategies $R = \{R_1,...,R_m\}$ and $S = \{S_1,...,S_n\}$ and payoff functions $A_i(q), i = 1,...,m$ and $B_j(p), j = 1,...,n$ that show interaction result for all strategies

Assume that *individuals of the 1st population interact only with individuals of secon population and vice versa: individuals of the 2nd population interact only with individuals of the 1st* .

At any moment of time *t* each individual uses a chosen strategy

Assume that

$$p(t) = (p_1(t),.., p_m(t)) \in \Delta^m = \left\{ p \in R_+^m \middle| \sum_i p_i = 1 \right\}$$

$$q(t) = (q_1(t),.., q_n(t)) \in \Delta^n = \left\{ q \in R_+^n \middle| \sum_j q_j = 1 \right\}$$

are population distributions over strategies.

Point $(p,q) \in \Delta^m \times \Delta^n$ is **Nash equilibrium of game Γ** if for any *i, j*

$$(p_i > 0) \Rightarrow i \in Arg \max_u B_u(q), (q_j > 0) \Rightarrow j \in Arg \max_u B_u(p)$$

Equilibrium is called **mixed** if for any *i, j* $p_i(t) < 1, q_j(t) < 1$

It is easy to see that

- For any game with indescrete payoff function there exist a Nash equilibrium
- In nonsingular case amount of positive coordinates *p* and *q* is the same

p(t) and *q(t)* change according to the system

$$\dot{p}_i = c(t, p(0), q(0)) G_i(p, A(q)), i = 1,\dots,m$$

$$\dot{q}_j = d(t, p(0), q(0)) H_j(q, B(p)), j = 1,\dots,n$$

This system is called Н-согласованной if

1) Functions G_i and H_j are such as

$$G_i(p, A) = 0, i = 1,\dots,m$$

$$H_j(q, B) = 0, j = 1,\dots,n$$

for any distributions $p \in \Delta^m, q \in \Delta^n$ and payoff vectors $A = (A_1,\dots, A_m), B = (B_1,\dots, B_n)$

so $\forall i (p_i > 0) \Rightarrow i \in Arg \max_u A_u ; \forall j (q_j > 0) \Rightarrow j \in Arg \max_v A_v$

That means that any Nash equilibrium is a stable point of system (4.1)

2) Functions *c* and *d* are measurable functions over *t* and continiously differentiable over *p(0)* and *q(0)*. Derivatives are equibounded over *t*.

3) Set $\Delta^m \times \Delta^n$ is invariant of system (4.1). Vector-functions *A, B, G and H* are continiously differentiable.

Note that MAIB, MAP and a system with positive functions ..generally correspond to these assumptions

Note that system (4.1) can be converted to autonomous system

$$\begin{aligned} \dot{p}_i &= G_i(p, A(q)), i = 1,\dots,m \\ \dot{q}_j &= H_j(q, B(q)), j = 1,\dots,n \end{aligned} \qquad (4.2)$$

if for any *t, p(0), q(0)*

$$\frac{c(p(0), q(0), t)}{d(p(0), q(0), t)} = \alpha(p(0), q(0))$$

This occasion takes place in interaction between populations with fixed sizes or between individuals with different roles in one population, for example, between 'owners' and 'invaders' (Maynard Smith, 1982)

Consider game Γ, system (4.1) and corresponding autonomous system (4.2)

●Stable point of system (4.2) is called **singular** if there is some eigenvalue λ of the Jacobian matrix that is equal to 0

●Point (p,a) is called a **centre** if for every eigenvalue Reλ=0, Imλ≠0

●Point (p,a) is called a **saddle** if there is an eigenvalue such that Reλ>0

Theorem 4.1: Any mixed equilibrium *(p*, q*)* is either a singular point, or a centre point, or a saddle point of (4.2)
in the latter case *(p*, q*)* is an unstable point of the system (4.1) for any functions *c, d.*

This theorem doesn't solve the question of stability for 'centre' points where all eigenvalues of the linearized matrix are purely imaginary. Let's use a method developed by Ritzberger and Vogelsberger (1990) and based on Liouville theorem.

Consider system $\dot{\pi}(t) = \varphi(\pi(t))$ According to this theorem, the field that is free of divergence ($div\varphi(\pi) \equiv 0$) keeps any volume constant and has noasymptotically stable points.

Let's describe MAIB class such that the given method permits to prove the abscence of asymptotically stable equilibria.

General MAIB equations for an interpopulation game are:

$$\dot{\pi}_j = -r_j \pi_j \sum_{i: f_i > f_j} q_{ji} \gamma_{ji} + \sum_{i: f_i < f_j} r_i \pi_i q_{ji} \gamma_{ji}, \; j \in J$$

Theorem 4.2: Assume that interpopulation MAIB is such that

1)r_j^k, γ_{ji}^k don't depend on $\pi^k (k=1,2)$ (the intensity of the pass to adaptive status and the probability of strategy change don't depend on distribution over strategies in the individual’s population. However, it can depend on distribution over strategies in the other population taking part in interacrion)

2) $q_{ji}^k = \pi_i^k (k=1,2)$ (the alternative strategy is chosen by means of random imtation of other individuals from the same population)

Then *any mixed equilibrium is not asymptotically stable*

- If the probability to choose a strategy as an alternative for individuals doesn't depend on the distribution over strategies in the same population then under general assumptions, the divergence of the vector field is determined by the right parts of the system is negative, and convergence to mixed equilibria is possible.

The problem of convergence to amixed equilibrium was considered in the literature also for iterative and continuous processes such as the fictitious play and Braun process for a game in normal form.

Denote $p^a(t), a \in A$, mixed strategies at stage *t*. Then **discrete Braun process is described by the system**:

- $p^a(t+1) = p^a(t)(1-t) + z^a(t)/t, a \in A, z \in Z_+$
- $z^a(t) \in Arg \max_{p^a \in P_0^a} u^a(p(t) \| p^a)$

where u^a is a payoff function of player *a* in mixed strategies

- P_0^a is a set of singular mixed strategies of player *a* that correspond to the set of his/her pure strategies

With respect to about behavior dynamics in the game of populations a $\in$ A, the process can be interpreted as adaptive in such way: *after every period t part 1/t of each population changes its strategy to one of the best replies'.*

- Braun conjectured and Robinson proved convergence of the discrete process to a mixed equilibrium for any zero-sum bimatrix game.
- Danskin showed convergence of this process for 2-person zero-sum games with continuous payoffs functions and compact sets of strategies
- Fudenberg and Kreps established convergence for nonantagonistic bimatrix game 2x2 with one pure mixed Nash equilibrium
- Benaim and Hirsch extended the result for 2x2 games with several Nash equilibria.
- In Bogdanov developed the approach by Belenkiy and proved that convergence is guaranteed for any bimatrix game that may be converted to zero-sum by means of transformation combination:
  - a) adding a constant to some column of the payoff matrix of the 1st player
  - b)adding a constant to same row of the payoff matrix of the 2nd player
  - c)multiplication of any payoff matrix by same positive constant
- It is known that those transformations keep the same set of Nash equilibria
- However, this result doesn’t hold for 2-person games in general

The fictitious play process doesn't converge for Shapley example, with payoff matrices:

$$A = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}, B = \begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$$

If pure strategies $(i_0, j_0) = (1,1)$ are taken as an initial point then the choice of strategies at times follows the cycle with six strategies:

$$(1,1) \to (1,3) \to (3,3) \to (3,2) \to (2,2) \to (2,1) \to (1,1)$$

The number of periods when the process is in each of these states grows increases exponentially in the number of rounds. Obviously the fictious play process doesn't converge.

For some nonantagonistic games, inparticular for Shapley example, the paths of the fictitious play process behave as the averages of RDM over time. It is shown that RDM is equivalent to MAIB particular case that corresponds to the theorem 4.2. At the same time not only the paths of these MAIB but also their averages over time don't converge to the equilibrium in Shapley example.

The best convergence to mixed equilibrium is provided by some more complex adaptive dynamics. Consider the following modification of indiscrete process of fictious play for the game of two players:

$$\begin{aligned}
\overset{\bullet}{q}{}^{1} &= \beta^{1}(q^{2}+\gamma\lambda(q^{2}-r^{2}))-q^{1} \qquad (4.3)\\
\overset{\bullet}{q}{}^{2} &= \beta^{2}(q^{1}+\gamma\lambda(q^{1}-r^{1}))-q^{2}\\
\overset{\bullet}{r}{}^{1} &= \lambda(q^{1}-r^{1})\\
\overset{\bullet}{r}{}^{2} &= \lambda(q^{2}-r^{2})
\end{aligned}$$

where q^a is reply of mixed strategy of player *a* at the moment of time.
$\beta^a(p^{-a})$ is the best reply of player *a* to partner's strategy p^{-a}
The idea is that $\lambda(q^a - r^a)$ approximates $\overset{\bullet}{q}{}^{a}$ when λ is quite big, i.e. the best reply is computed for the future strategy.

For Shapley example $0{,}0413<\gamma/(1-\gamma)<0{,}0638$ is sufficient for local stability of the equilibrium for system (4.3)
One more direction of interest in this field is gradient dynamic models where the mixed strategy of each player changes to the direction of his payoff function gradient. It is shown that if the gradient is computed for the strategy of partner at the current moment, then Nash equilibrium is never locally stable for the system. However, if the gradient is computed for the future partner's strategy defined as in the case of the fictitious play then a relevant choice of γ provides stability.

About evolution of altruism and cooperation

●Does the principle of individual fitness maximization correspond to actual behavior in biological and social populations?

●What refinements should be implemented to the model for more precise reflection of behavior evolution ?

First, consider behavior in biological populations. A general opinion of biologists is that, on the whole, the principle of individual fitness maximization does not contradict with actual behavior (see III Congress of the ESEB, 1991). Exceptions: **altruistic** and **cooperative** behavior.
The concepts of altruistic and cooperative behavior may be illustrated by different variants of the ‘*Prisoner's dilemma*’. In this symmetric two-player game each player has two possible strategies: to cooperate *(C)* or to be selfish *(S)*.

In the general Prisoner’s dilemma, given any behavior of the other player, the selfish strategy is more profitable $(u_{ss} > u_{cs}, u_{c\kappa} > u_{cc})$ while at the same time the total gain is maximal when both players cooperate $(u_{cc} > (u_{cs} + u_{sc})/2, u_{cc} > u_{ss})$

Consider the following payoff matrix

$$\begin{array}{ccc} & C & S \\ C & (5,5) & (1,6) \\ S & (6,1) & (2,2) \end{array}$$

In this game there exists a unique Nash equilibrium, which corresponds to selfish behavior and is also a dominance solution. However, in actual Prisoner’s dilemma-like situations, players often cooperate.

Altruistic behavior deviates even more from individual fitness maximization. Consider the following payoff matrix (*A* denotes altruistic behavior, *S* denotes selfish behavior):

$$\begin{array}{ccc} & A & S \\ A & (5,5) & (1,10) \\ S & (10,1) & (2,2) \end{array}$$

Here, altruistic behavior by one player combined with selfish behavior by the other player corresponds to total fitness maximization. Meanwhile, the altruist in such an outcome obtains less than his guaranteed payoff, which he could get under his Nash equilibrium strategy.

As an **example of cooperative behavior** in biological populations, let us note the behavior of animals that take turns standing guard, or predators that participate in joint hunting. Guard gives a signal when predator is coming. Profitable deviation:

- Don't safeguard and don't give a signal
- Safeguard and don't give a signal

Giving signal attracts predator attention to the guard. If no one safeguards than everybody loose.

Another example is joint hunting. Selfish players save energy in prejudice of everybody.

Examples of altruism are typical in relatives interaction.

- *The altruistic behavior of parents towards children*
  - is rather widespread and does not contradict with the concept of fitness maximization, since fitness is equal to the sum of fertility and viability.
  - If individual saves his offspring at predator's expense then it is optimal from the fitness point of view

- Interesting examples of *altruism are those that do not relate to individual fitness maximization*
  - behavior of social insects (bees, ants, termits)
  - Individuals in families don't maximize their individual fitness because they have no offspring. But they are fearlessly attacking any predator protecting their family
  - The explanation is that individuals in insect families are close relatives.

A shortcoming of the model of direct inheritance is that it takes into account only the relative type "parent-child" and does not consider relations between siblings, cousins, etc. Taking these relations in account, it is possible to explain the spread of cooperative and altruistic behavior in the sense of the total fitness maximization for the group of relatives.

Let us describe the corresponding model. Assume that interaction in a population is characterized by the set of strategies *S* and fitness functions $f_i(\pi), s \in S$. In contrast to the previous models, **individuals can distinguish between siblings** (“sibs”), that is brothers and sisters, and other members of the population and choose a strategy depending on this characteristic.

Thus, a full strategy *(s,s’)* includes

- component *s* which is applied to sibs
- *s’* for other individuals (strangers)

At a time period, an individual interacts with sibs with some frequency $\lambda \in (0,1)$ and with strangers with frequency $1\text{-}\lambda$. Total fitness additively depends on the results of interactions with relatives and the rest of the population:

$$\overline{f}_{(s,s')}(\pi') = \lambda_r f_s^r(s) + (1-\lambda_r) f_{s'}(\pi')$$

where

- f_s^r ,$f_{s'}$ determine the results of the interaction respectively with relatives and strangers
- π’ is a distribution over component *s’*
- We assume that all sibs play the same strategies.

Thus, the interaction is characterized by a population game

$$\overline{G} = \left\langle \overline{S} = \{(s, s') \in S \times S\}, \overline{f_{s,s'}}(\overline{\pi}) = \lambda_r f_s^r(s) + (1 - \lambda_r) f_{s'}(\pi') \right\rangle$$

where $\overline{\pi}$ is a distribution over full strategies.

Theorem 5.1 Any strategy *(s,s')* where $s \notin Arg\max_i f_i^r(i)$ is strictly dominated by strategy *(s*,s')* where $s^* \in Arg\max_i f_i(i)$ Distribution π is a Nash equilibrium if for all specified non-optimal strategies $\pi_{ss'} = 0$ and the corresponding distribution *π'* is a Nash equilibrium of the game $\langle S, f_{s'}(\pi') \rangle$

Thus, in any conflict similar to a Prisoner's dilemma, sibs play the cooperative strategy with respect to each other.

In order to explain altruistic behavior, consider the following modification of the model.
Assume that in the interaction sibs may take on one of two different roles (dominating and subordinating). Let the strategy and the fitness function depend on the role. Then the full strategy with respect to relatives is determined by the pair $s = (s_\alpha, s_\beta) \in S \times S$

Only strategies $s^* = (s^*_\alpha, s^*_\beta) \to \max\limits_{(s_\alpha, s_\beta)} (f^\alpha(s) + f^\beta(s))$ which provide the maximum total fitness, survive elimination of strictly dominated strategies. Proceeding from Theorem 5.1, we may conclude that evolution in self-reproducing populations leads to behavior that maximizes the total fitness of sibs. Though the last model contains an implicit restriction on the evolutionary mechanism that determines the distribution over strategies with respect to relatives, this restriction is not important: any other mechanism would lose in competition with the given mechanism, which realizes the optimal strategy s^* specified in theorem 5.1.

Note that these results may be generalized for relations between cousins, second cousins, etc.

- Full strategy includes variants of behavior *s1, ..., sk* with respect to relatives of different degrees *1,...,k* and *s'* with respect to strangers.
- The fitness function is of the form $f_s(\pi)=\sum \lambda_i f_{si}^i(si)+(1-\sum \lambda_i)f_{s'}(\pi'), \lambda_i \in (0,1)$, λ characterizes the frequency of interaction with relatives of degree *i*.

As above, we assume that all relatives apply the same strategy with respect to each other. This assumption corresponds to the model of direct inheritance and does not restrict generality within the context of the proposed model of evolutionary mechanism selection. Then, elimination of strictly dominated strategies leads to optimization of relations between relatives: surviving strategies are those s^* such that $si^* \in Arg\max f_{si}^i(si), i=1,...,k$

As in biological, in social populations with low migration level interacting individuals usually have common ancestors *at least in 7th generation*

- Consider two random individuals from population with 10^5 individuals
- Assume that they don't have common ancestors in 7 generations
- Then, 7 generations ago each of them had 64 ancestors of different sex
- Let's take 104 as the size of the population (that means at average 4 children in a family)
- Condition that no one ancestor female of one individual married male ancestor of another individual is necessary condition for having no common ancestors
    - Probability of this event: (1-64/5900)129 < 0,1

Proceeding from those results, we should expect widespread cooperative and altruistic behavior for total fitness maximization. However real behavior doesn't usually correspond to friendship principles. And in biological populations such behavior forms are not dominating. Examples of tough competition between relatives are well-known. Sometimes they even eat children of each other. One of reasons for limited spreading of cooperative behavior is its **insecurity in case of implemention of selfish mutants.**

Relating to social populations, there exists one more problem. Density of relationship inside a family is specific for different nations. So, it seems interesting to define *degrees of altruism and cooperation in Nash equilibrium depending on these parametres and mutation intensity.*

Note that the problem of cooperative and altruistic behavior prevalence is studied in different settings.

For instance, the theory of repeated games explains the prevalence of cooperation in repeated conflict situations proceeding from individual fitness optimization. Repetition provides the possibility to punish those individuals who fail to behave cooperatively. Taking into account such future punishments, cooperative behavior turns out to be individually profitable. While the equivalence between individual fitness and total fitness does not hold in the evolutionary models considered above, cooperative behavior nevertheless can spread broadly due to the selection of evolutionary mechanisms.

Peculiarities of behavior evolution in social populations.
Superindividuals.
Their impact on utility functions and processes of nation reproduction

Actual behavior in modern social populations maximizes neither individual nor group fitness. Let us take social-welfare states such as Sweden or Germany. Favorable conditions for survival and normal biological development are guaranteed there to any newborn citizen regardless of his or her social origin. However, demographic statistics indicate that most people ignore the advantages in reproductive opportunities. At the end of the twentieth century GNP per head of population in Germany and Sweden was 10 times biger then in Russia. The fertility rate in those countries was identically low: 9-10 per 1000. For instance, the pool of Stockholm inhabitants in 1995 showed that 70% of the adult population had no children and did not plan to have them. («World population prospects», 1996)

Why do the evolutionary models examined above not seem to apply directly for current social populations? Let us note factors that explain this discrepancy.

a)the impossibility for children to reliably inherit parental strategies within social populations.

The behavior strategies are so complicated and the environment is so variable that an individual could spend most of his life trying to teach his descendants. The division of labor and differential access to educational institutions appeared at an early stage of mankind's development. These institutions play a crucial role in forming the behavior of new generations. Later governments, churches and other organizations realized the importance of educational institutions and effectively used them to form desirable behavior.

b)The problem of payoff evaluation for different behavior strategies is rather complicated even for theoretical analysis.

In practice, decision-makers are usually unable to carry out a complete evaluation in a reasonable time. Biological evolution has formed various mechanisms that facilitate fast decisions that generally are close to though not completely optimal in the sense of fitness.

One such mechanism is the **feeling of pleasure or satisfaction** related to food and comfort.

In nature, actions that are pleasant or conducive to achieving pleasure are usually rational in the sense of individual reproduction. In particular, provision and consumption of food and some other resources are necessary for reproduction. However, devotion to this objective function may impede reproduction in some instances. One example of fitness-impeding commitment to food consumption within an ecological system is the interaction between a lamehuza beetle and ants who feed on secretions produced by the beetle. Sometimes the secretions have a drug-like effect on ants: they throw the queen out of the ant-hill, put the lamehuza beetle in the queen’s place and take care of it in order to get more secretions. In time this ant family perishes.

This example is exceptional for ecosystems. The situation in modern human society is different. A lot of people consume large amounts of alcohol, tobacco, excessive food and other goods that are harmful or at least unnecessary for reproduction. There are many families and single individuals spending the whole of their lives to earn money for such consumption and finally having at most one child.

Another auxiliary mechanism of strategy determination is **leader imitation**.

- In natural populations it promotes learning and permits the coordination of actions of individuals within groups and thus increases their fitness.
- In social populations imitation creates an additional possibility for behavior manipulation by choosing an appropriate leader.

The methods of behavior manipulation described above (control over the educational process, usage of pleasure incentives and the imitation mechanism) have been practiced since ancient times. But the situation has dramatically changed with the development of mass media in the 20th century. Currently, television provides the possibility to influence billions of people by playing the role of a teacher and creating models for imitation.

Now consider who or what changes the objective functions in order to form certain behavior in social populations.

- In our previous examples for ecosystems, we identified two different variants. In the case of the ants and the lamehuza beetle, the manipulator is an individual belonging to another population. The interaction between the populations is of the "predator-prey" type.
- In the case of social insects, individual behavior is formed by a self-reproducing superindividual – the family of ants, bees or other social insects. While suppressing individual reproduction of some part of the population, the evolutionary mechanism provides efficient reproduction of these superindividuals and the population as a whole.

In general a superindividual related to a given biological or human population is a self-reproducing structure that includes some individuals of this population among its elements. Besides the individual organisms, the superindividual may comprise other material or immaterial components. In social populations we meet similar and more complicated variants of behavior regulation. In addition to superindividuals of a biological nature (families), there exist self-reproducing superindividuals of a social-economic nature (corporations, art and scientific schools, public and religious organizations, and government institutions). Instead of replication, superindividuals may either grow, involving new human and material resources, or collapse.
Thus, superindividuals use the population as a resource for their own reproduction and growth, and influence population behavior for this purpose. The more intensive is the specific activity of involved individuals, the faster is growth and development of the superindividual. Since the time and energy of each person is limited, the superindividual often suppresses other kinds of activity, in particular, individual reproduction.

According to all summarized results it is possible to formulate hypotesis that individual utility functions in modern society are generally determined by superindividuals who form them in a way to provide this reproduction and growth. In other words, dynamic models of competition and selection is better to formulate and use at superindividual level.

Analysis of such models connected to competition gives us useful results from endogenuos determination of payoff functions point of view. For research of other social-economic processes it is good to use models of interaction and natural selection of self-reproducing individuals of different types. According to complexity of social systems it seems impossible to build a complete quantified evolution model of social behavior that will be similar to replicator dynamic model. Nevertheless, conception of superindividual competition and their influence on social behavior helps to analyse some problems of economics and sociology. In particular, this conception helps to produce a human behavior model that penetrates disadvantages of 'economic man' and 'biologic man'.

Let's develop an approach proposed by Germeyer in 1973.

●Individual strategies (chosen conciously or not) are distributions of his/her time and other resources to different activities

●This distribution s^a and noncontrolled factors z determine the values of auxiliary utility function $.u_0^a$ That shows satisfaction from consumption and also reproduction parametres:

1) for himself (survival u_1^a)
2) for his family (fitness u_2^a)
3) for different superindividuals he is included to (parametres $u_j^a, j = 3,4,\ldots$)

●Choosing strategy s^a he maximizes some aggregate of these functions

●In mentioned approach the criterion $\min_i(\frac{u_i^a(s^a,z) - w_i^a(z)}{\lambda^a})$ is considered. It can be interpreted in such way:

- w_i^a is a minimal possible value of component i
- λ_i^a is importance of maximizating this component according to minimal possible level

According to this criteria individual spends his resourse on maximization of the worst indexes. Real behavior is not always like this criterion. Quite often individual uses resources in a way where he supposes the most effect (formally it corresponds to changing minimum and maximum in this equation). Different people are characterised with different types of aggregation and different indexes.

Let's note similarities of this model with L. Gumilev concept. He separates the following types:

1) Harmonious people are the individuals who work as much as it is necessary for life sustaining and offspring liмes. Formally this type maximizes combination of individual and family fitness.

2) Subpassionarities can't control their wills even if their satisfaction does harm to themselves and people around them. Do not take care of offsprings. Maximize u_0^a .

3)Passionaries are individuals with hanged energy. Their work is changing environment. Their activity is not connected with material benefits, pleasures, individual or family reproduction. Workaholics providing manufacturing and economical corporation success, eggective work of government institutions, scientists, artists who develop their area of ineterst despite material profits.

From social and economic modelling point of view revealing of harmonious, subpassionarities and passionarities people distribution over self-reproducing structures of different types is of great interest. Collecting and processing suitable information is a challenging problem in sociology and experimental economics.

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